A Comparison of the Performance and Power Requirements for Switchable Damper and Active Suspension Systems

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***Abstract*— In this paper, the full-car model for passive suspension system (PSS), switchable damper suspension system (SDSS) and active suspension system (ASS) are compared in terms of their relative power requirements and ride performance. The linear quadratic regulator (LQR) and Fuzzy logic control (FLC) strategies are used for the system behavior and compared relative to the PSS. The PSS, SDSS, and ASS are evaluated in terms of ride performance criteria. The optimal suspension parameters values are evaluated. The results revealed that the ASS with the LQR control strategy gives better ride performance compared with PSS and SDSS. The ASS with the FLC strategy gives the best ride performance compared with the ASS using LQR control, and SDSS with LQR and FLC strategies. The mean power demand ($\overline{\mathrm{P}}_{\mathrm{Dem}}$) and dissipation ($\overline{\mathrm{P}}_{\mathrm{Diss}}$) within the suspension systems are evaluated and discussed.**

***Index Terms*— Active suspension system (ASS), Fuzzy control (FLC), LQR control, Ride performance, Switchable damper (SD).**

I. INTRODUCTION

THE ASS is one in which a hydraulic or pneumatic actuator is utilized in the suspension system in conjunction with or as a replacement for the PSS. It is evident that ASS possess considerable potential for improving vehicle ride and handling compared with other intelligent suspension systems [1-2]. In spite of the performance results of the theoretical ASS studies, its utilization is limited to some prototype vehicles due to its increased cost, complexity coupled with high energy consumption [3-4]. Soliman et al [5] developed a mathematical model for the twin spring system using a half-car model for PSS to study the effect of front and rear spring stiffness on the vehicle dynamics. Their results revealed that the measured values of the vertical acceleration and suspension working space were 8% to 10% higher than those predicted. Giliome et al [6] concerned their study with the development of a semi-active hydro-pneumatic spring and damper system to enhance the ride comfort and handling. A test-rig of single degree of freedom (DOF) with a body mass of 3 tons was used. They concluded that by utilizing the semi-active hydro-pneumatic spring with a high and low spring rate a suspension system that is optimized for both handling and ride comfort can be achieved. Heo et al [7] concerned their study with the continuously variable damper; the impact of the damper characteristics changes upon ride comfort and driving safety have been examined via simulation. Their results indicated that the soft damping force limit affects the performance of the semi-active suspension system (SASS) more than the hard limit. Hence the design parameters that affect the soft damping limit must get more attention than others.

Soliman [8] analyzed the effect of the SDSS on vehicle ride quality control. The simulation results indicated that the SDSS with adaptive control is superior compared with the PSS. An improvement of 20% of the root mean square (RMS) of vehicle vertical acceleration is achieved using the SDSS with adaptive control. The power dissipation ($\mathrm{P}_{\mathrm{Diss}}$) in suspension relative to power consumed in rolling resistance for the SDSS is discussed. Other researchers studied the effect of SDSS, SASS, and ASS on the vehicle ride performance [9-14], however, their research concentrated on the linear optimal control theory and adaptive control using a quarter or half-vehicle model. In this paper, the LQR and FLC strategies are used for the system's behavior using the full-car model. Also, the $\overline{\mathrm{P}}_{\mathrm{Dem}}$ and $\overline{\mathrm{P}}_{\mathrm{Dem}}$ within the suspension systems are assessed and discussed.

II. MATHEMATICAL MODEL

1) *Three-setting SDSS Model*

The SDSS for a full-car model is shown in Fig. 1.

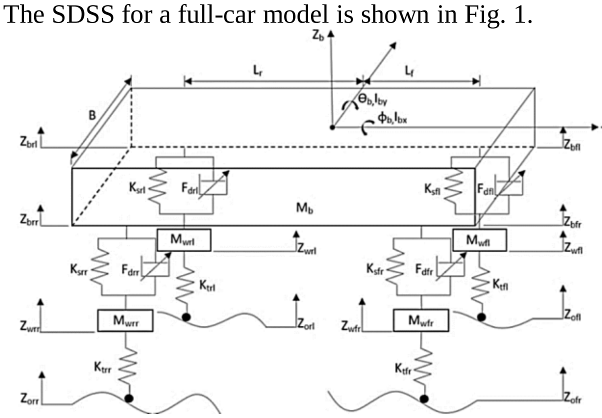


Fig. 1. SDSS for full-car model

The equations of motion (EOM) can be written in the matrix form as follows;

$$
\begin{bmatrix}
M_b & 0 & 0 & 0 & 0 & 0 & 0\\
0 & I_{bx} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & I_{by} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & M_{wfr} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & M_{wfl} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & M_{wrr} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & M_{wrl}
\end{bmatrix}
\begin{bmatrix}
\ddot{z}_b\\ \ddot{\phi}_b\\ \ddot{\theta}_b\\ \ddot{z}_{wfr}\\ \ddot{z}_{wfl}\\ \ddot{z}_{wrr}\\ \ddot{z}_{wrl}
\end{bmatrix}
$$

$$
+\begin{bmatrix}
c_{sfl}+c_{sfr}+c_{srl}+c_{srr} & \frac{B}{2}.c_{sfl}-\frac{B}{2}.c_{sfr}+\frac{B}{2}.c_{srl}-\frac{B}{2}.c_{srr} & \dots\\
\frac{B}{2}.c_{sfl}-\frac{B}{2}.c_{sfr}+\frac{B}{2}.c_{srl}-\frac{B}{2}.c_{srr} & \left(\frac{B}{2}\right)^2 c_{sfl}+\left(\frac{B}{2}\right)^2 c_{sfr}+\left(\frac{B}{2}\right)^2 c_{srl}+\left(\frac{B}{2}\right)^2 c_{srr} & \dots\\
-L_f.c_{sfl}-L_f.c_{sfr}+L_r.c_{srl}+L_r.c_{srr} & -\frac{B}{2}L_f.c_{sfl}+\frac{B}{2}L_f.c_{sfr}+\frac{B}{2}L_r.c_{srl}-\frac{B}{2}L_r.c_{srr} & \dots\\
-c_{sfr} & \frac{B}{2}.c_{sfr} & \dots\\
-c_{sfl} & -\frac{B}{2}.c_{sfl} & \dots\\
-c_{srr} & \frac{B}{2}.c_{srr} & \dots\\
-c_{srl} & -\frac{B}{2}.c_{srl} & \dots
\end{bmatrix}
$$

$$
\begin{matrix}
-L_f.c_{sfl}-L_f.c_{sfr}+L_r.c_{srl}+L_r.c_{srr} & -c_{sfr} & -c_{sfl} & -c_{srr} & -c_{srl}\\
-\frac{B}{2}L_f.c_{sfl}+\frac{B}{2}L_f.c_{sfr}+\frac{B}{2}L_r.c_{srl}-\frac{B}{2}L_r.c_{srr} & \frac{B}{2}.c_{sfr} & -\frac{B}{2}.c_{sfl} & \frac{B}{2}.c_{srr} & -\frac{B}{2}.c_{srl}\\
L_f^{\,2}.c_{sfl}+L_f^{\,2}.c_{sfr}+L_r^{\,2}.c_{srl}+L_r^{\,2}.c_{srr} & L_f.c_{sfr} & L_f.c_{sfl} & -L_r.c_{srr} & -L_r.c_{srl}\\
L_f.c_{sfr} & c_{sfr} & 0 & 0 & 0\\
L_f.c_{sfl} & 0 & c_{sfl} & 0 & 0\\
-L_r.c_{srr} & 0 & 0 & c_{srr} & 0\\
-L_r.c_{srl} & 0 & 0 & 0 & c_{srl}
\end{matrix}
\Bigg]
\begin{bmatrix}
\dot{z}_b\\ \dot{\phi}_b\\ \dot{\theta}_b\\ \dot{z}_{wfr}\\ \dot{z}_{wfl}\\ \dot{z}_{wrr}\\ \dot{z}_{wrl}
\end{bmatrix}
$$

$$
+\begin{bmatrix}
k_{sfl}+k_{sfr}+k_{srl}+k_{srr} & \frac{B}{2}.k_{sfl}-\frac{B}{2}.k_{sfr}+\frac{B}{2}.k_{srl}-\frac{B}{2}.k_{srr} & \dots\\
\frac{B}{2}.k_{sfl}-\frac{B}{2}.k_{sfr}+\frac{B}{2}.k_{srl}-\frac{B}{2}.k_{srr} & \left(\frac{B}{2}\right)^2 k_{sfl}+\left(\frac{B}{2}\right)^2 k_{sfr}+\left(\frac{B}{2}\right)^2 k_{srl}+\left(\frac{B}{2}\right)^2 k_{srr} & \dots\\
-L_f.k_{sfl}-L_f.k_{sfr}+L_r.k_{srl}+L_r.k_{srr} & -\frac{B}{2}L_f.k_{sfl}+\frac{B}{2}L_f.k_{sfr}+\frac{B}{2}L_r.k_{srl}-\frac{B}{2}L_r.k_{srr} & \dots\\
-k_{sfr} & \frac{B}{2}.k_{sfr} & \dots\\
-k_{sfl} & -\frac{B}{2}.k_{sfl} & \dots\\
-k_{srr} & \frac{B}{2}.k_{srr} & \dots\\
-k_{srl} & -\frac{B}{2}.k_{srl} & \dots
\end{bmatrix}
$$

$$
\begin{matrix}
-L_f.k_{sfl}-L_f.k_{sfr}+L_r.k_{srl}+L_r.k_{srr} & -k_{sfr} & -k_{sfl} & -k_{srr} & \dots\\
-\frac{B}{2}L_f.k_{sfl}+\frac{B}{2}L_r.k_{srl}+\frac{B}{2}L_f.k_{sfr}-\frac{B}{2}L_r.k_{srr} & \frac{B}{2}.k_{sfr} & -\frac{B}{2}.k_{sfl} & \frac{B}{2}.k_{srr} & \dots\\
L_f^{\,2}.k_{sfl}+L_f^{\,2}.k_{sfr}+L_r^{\,2}.k_{srl}+L_r^{\,2}.k_{srr} & L_f.k_{sfr} & L_f.k_{sfl} & -L_r.k_{srr} & \dots\\
L_f.k_{sfr} & k_{sfr}+k_{tfr} & 0 & 0 & \dots\\
L_f.k_{sfl} & 0 & k_{sfl}+k_{tfl} & 0 & \dots\\
-L_r.k_{srr} & 0 & 0 & k_{srr}+k_{trr} & \dots\\
-L_r.k_{srl} & 0 & 0 & 0 & \dots
\end{matrix}
$$

$$
\begin{matrix}
-k_{srl}\\ -\frac{B}{2}.k_{srl}\\ -L_r.k_{srl}\\ 0\\ 0\\ 0\\ k_{srl}+k_{trl}
\end{matrix}
\Bigg]
\begin{bmatrix}
z_b\\ \phi_b\\ \theta_b\\ z_{wfr}\\ z_{wfl}\\ z_{wrr}\\ z_{wrl}
\end{bmatrix}
=
\begin{bmatrix}
F_{sfr}+F_{sfl}+F_{srr}+F_{srl}\\
\frac{B}{2}\left(F_{srl}+F_{sfl}\right)-\frac{B}{2}\left(F_{srr}+F_{sfr}\right)\\
-L_f\left(F_{sfr}+F_{sfl}\right)+L_r\left(F_{srr}+F_{srl}\right)\\
k_{tfr}.z_{ofr}\\
k_{tfl}.z_{ofl}\\
k_{trr}.z_{orr}\\
k_{trl}.z_{orl}
\end{bmatrix}
\tag{1}
$$

2) *ASS Model*

The ASS for a full-car model is shown in Fig. 2.

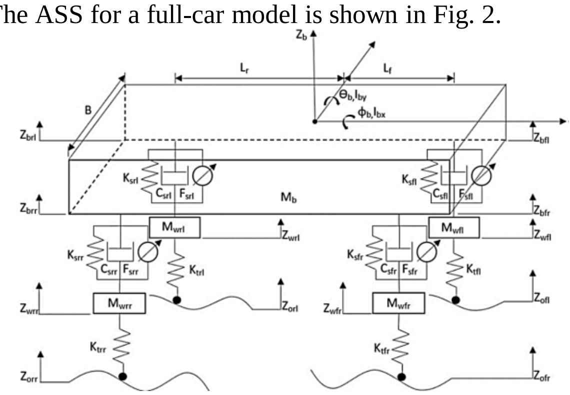


Fig. 2. The ASS for full-car model

The EOM can be written in the matrix form as follows;

$$
\begin{bmatrix}
M_b & 0 & 0 & 0 & 0 & 0 & 0\\
0 & I_{bx} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & I_{by} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & M_{wfr} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & M_{wfl} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & M_{wrr} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & M_{wrl}
\end{bmatrix}
\begin{bmatrix}
\ddot{z}_b\\ \ddot{\phi}_b\\ \ddot{\theta}_b\\ \ddot{z}_{wfr}\\ \ddot{z}_{wfl}\\ \ddot{z}_{wrr}\\ \ddot{z}_{wrl}
\end{bmatrix}
$$

$$
+\begin{bmatrix}
k_{sfl}+k_{sfr}+k_{srl}+k_{srr} & \frac{B}{2}.k_{sfl}-\frac{B}{2}.k_{sfr}+\frac{B}{2}.k_{srl}-\frac{B}{2}.k_{srr} & \dots\\
\frac{B}{2}.k_{sfl}-\frac{B}{2}.k_{sfr}+\frac{B}{2}.k_{srl}-\frac{B}{2}.k_{srr} & \left(\frac{B}{2}\right)^2 k_{sfl}+\left(\frac{B}{2}\right)^2 k_{sfr}+\left(\frac{B}{2}\right)^2 k_{srl}+\left(\frac{B}{2}\right)^2 k_{srr} & \dots\\
-L_f.k_{sfl}-L_f.k_{sfr}+L_r.k_{srl}+L_r.k_{srr} & -\frac{B}{2}L_f.k_{sfl}+\frac{B}{2}L_f.k_{sfr}+\frac{B}{2}L_r.k_{srl}-\frac{B}{2}L_r.k_{srr} & \dots\\
-k_{sfr} & \frac{B}{2}.k_{sfr} & \dots\\
-k_{sfl} & -\frac{B}{2}.k_{sfl} & \dots\\
-k_{srr} & \frac{B}{2}.k_{srr} & \dots\\
-k_{srl} & -\frac{B}{2}.k_{srl} & \dots
\end{bmatrix}
$$

$$
\begin{matrix}
-L_f.k_{sfl}-L_f.k_{sfr}+L_r.k_{srl}+L_r.k_{srr} & -k_{sfr} & -k_{sfl} & -k_{srr} & \dots\\
-\frac{B}{2}L_f.k_{sfl}+\frac{B}{2}L_r.k_{srl}+\frac{B}{2}L_f.k_{sfr}-\frac{B}{2}L_r.k_{srr} & \frac{B}{2}.k_{sfr} & -\frac{B}{2}.k_{sfl} & \frac{B}{2}.k_{srr} & \dots\\
L_f^{\,2}.k_{sfl}+L_f^{\,2}.k_{sfr}+L_r^{\,2}.k_{srl}+L_r^{\,2}.k_{srr} & L_f.k_{sfr} & L_f.k_{sfl} & -L_r.k_{srr} & \dots\\
L_f.k_{sfr} & k_{sfr}+k_{tfr} & 0 & 0 & \dots\\
L_f.k_{sfl} & 0 & k_{sfl}+k_{tfl} & 0 & \dots\\
-L_r.k_{srr} & 0 & 0 & k_{srr}+k_{trr} & \dots\\
-L_r.k_{srl} & 0 & 0 & 0 & \dots
\end{matrix}
$$

$$
\begin{matrix}
-k_{srl}\\ -\frac{B}{2}.k_{srl}\\ -L_r.k_{srl}\\ 0\\ 0\\ 0\\ k_{srl}+k_{trl}
\end{matrix}
\Bigg]
\begin{bmatrix}
z_b\\ \phi_b\\ \theta_b\\ z_{wfr}\\ z_{wfl}\\ z_{wrr}\\ z_{wrl}
\end{bmatrix}
=
\begin{bmatrix}
F_{dfr}+F_{dfl}+F_{drr}+F_{drl}\\
\frac{B}{2}\left(F_{drl}+F_{dfl}\right)-\frac{B}{2}\left(F_{drr}+F_{dfr}\right)\\
-L_f\left(F_{dfr}+F_{dfl}\right)+L_r\left(F_{drr}+F_{drl}\right)\\
k_{tfr}.z_{ofr}\\
k_{tfl}.z_{ofl}\\
k_{trr}.z_{orr}\\
k_{trl}.z_{orl}
\end{bmatrix}
\tag{2}
$$

III. Road Input and Vehicle Parameters

The road profile used is shown in Fig. 3. The study was conducted at a distance of 1 km with a car speed of 25 m/s and a study duration of 40 sec. The delay between the front and rear wheel is calculated as follows:

$$
\Delta t = \frac{L \text{ "Wheelbase"}}{U \text{ "Vehicle speed"}} \tag{3}
$$

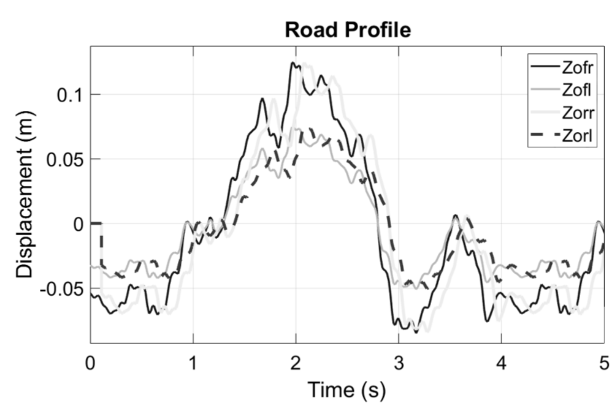


Fig. 3. The road profile

The road roughness coefficient is $3{*}10^{-6}$ for the right track and the left track is 60% of it, this represents a road that is somewhat worse than average main road. The slope of road input is 2.5 [14].

The data used in this work for the full-car model are listed in Table I. These refer to a full-car of large saloon station [13].

TABLE I
VEHICLE PARAMETERS

Symbol	Description	Units
M_b	Body mass	1350 Kg
M_{wfr} or M_{wfl}	Front right or left wheel mass	40.5 Kg
M_{wrr} or M_{wrl}	Rear right or left wheel mass	45.4 Kg
I_{by}	Pitch moment of inertia	2400 Kg.m^2
I_{bx}	Roll moment of inertia	400 Kg.m^2
K_{sfr} or K_{sfl}	Front right or left spring stiffness	20000 N/m
K_{srr} or K_{srl}	Rear right or left spring stiffness	22000 N/m
C_{sfr} or C_{sfl}	Front right or left damper coefficient	1600 N.s/m
C_{srr} or C_{srl}	Rear right or left damper coefficient	1800 N.s/m
K_{tfr} or K_{tfl}	Front right or left damper coefficient	190000 N/m
K_{trr} or K_{trl}	Rear right or left tire spring stiffness	190000 N/m
L_f	Distance from front axle to c.g.	1.25 m
L_r	Distance from rear axle to c.g.	1.5 m
L	Wheelbase, L	2.75 m
B	Wheel track, B	1.5 m

IV. CONTROL STRATEGIES

1) *FLC Model and Rules for SDSS*

Fuzzy Simulink model for SDSS is shown in Fig. 4. The fuzzy algorithm deals with sets that have fuzzy boundaries. Fuzzy logic is multi-valued; it deals with degrees of membership and degrees of truth. Also, it deals with nonlinear, uncertain, or imprecise decision-making problems [15]. In this case, the vehicle body velocity and relative velocity between wheels and body are the inputs for the controller, and the control force that represents the damping force is the output. Table II shows the rules used by the Mamdani fuzzy inference system (FIS) for the SDSS. The rules evaluation surface and MFs for the SDSS FLC are shown in Fig. 5 and 6 [16].

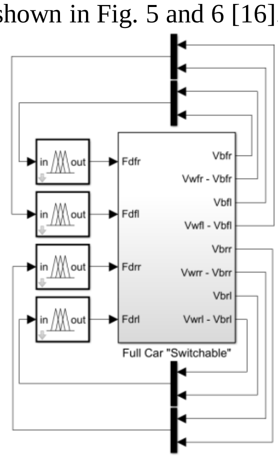


Fig. 4. Fuzzy Simulink model for SDSS

TABLE II
7×7 RULE TABLE FOR SDSS

Body Velocity	Relative Velocity: P	Relative Velocity: Z	Relative Velocity: N
P	L	M	S
Z	M	M	M
N	S	M	L

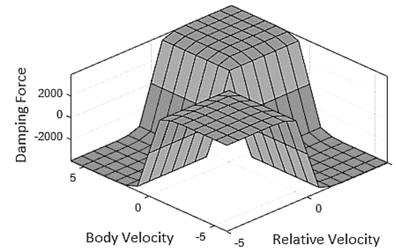
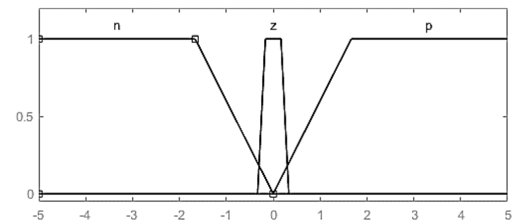
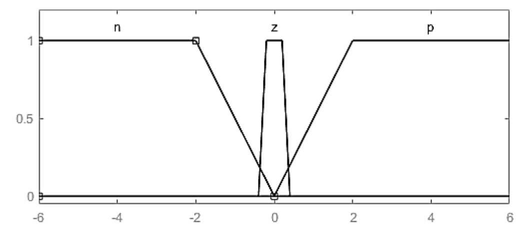


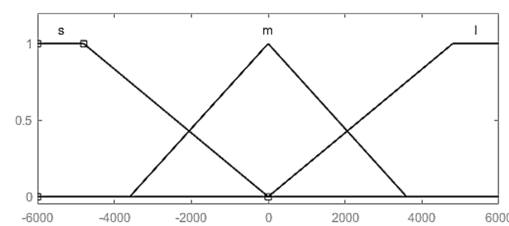
Fig. 5. Rules evaluation surface for SDSS



(a)



(b)



(c)

Fig. 6. MFs for the SDSS FLC, (a) Velocity, (b) Acceleration, (c) Damping Force

2) *Fuzzy Rules for ASS*

Fuzzy Simulink model for ASS is shown in Fig. 7. The FLC approach is applied to the full car model problem using two separate FLC solving two Multi Input Single Output (MISO) system problems, one for the front and the other for the rear. In this case, the vehicle body velocity and acceleration are the inputs for the controller, and the control force that represents the actuator force is the output. Table III shows the rules used by the FIS to take the proper decisions [17] and Fig. 8 representing all rules evaluation probabilities as a surface.

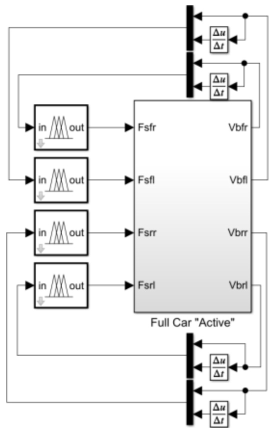


Fig. 7. Fuzzy Simulink model for ASS

TABLE III
3× 3 RULE TABLE FOR THE ASS

Velocity \ Acceleration	**P**	**Z**	**N**
P	N	N	Z
Z	N	Z	P
N	Z	P	P

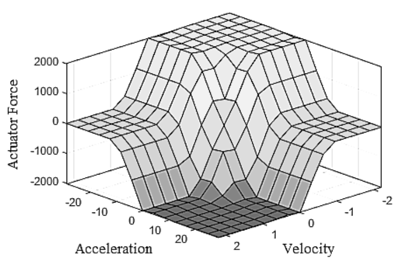
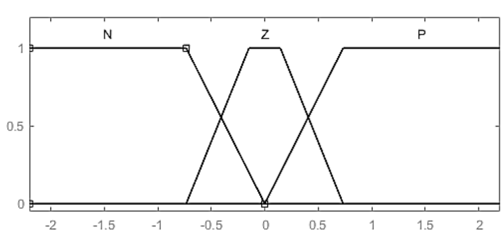
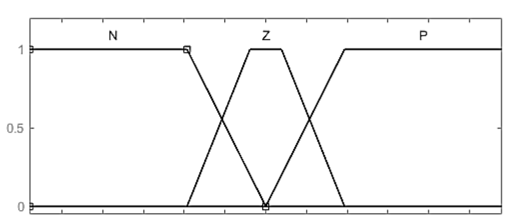


Fig. 8. Rules evaluation surface for the ASS

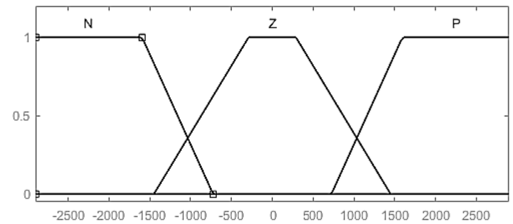
The MFs used for the inputs and outputs are shown in Fig. 9. A trapezoidal Membership Functions (MFs) are used as they present a smoother control action and evaluation as proposed by Gandhi [17].



(a)



(b)



(c)

Fig. 9. MFs for the ASS FLC, (a) Velocity, (b) Acceleration, (c) Actuator Force

3) *LQR for SDSS*

Considering the system under study as a linear time-invariant system that has a state-space model as follows:

$$\dot{X} = A.X + B.U + D.W \tag{4}$$

$$Y = C.X \tag{5}$$

Fig. 10 shows the LQR Simulink model for the SDSS. Taking the state variables as follows:

$$X = [\dot{z}_{bfl} \quad \dot{z}_{bfr} \quad \dot{z}_{wfl} \quad \dot{z}_{wfr} \quad \dot{z}_{brl} \quad \dot{z}_{brr} \quad \dot{z}_{wrl} \quad \dot{z}_{wrr} \quad \ldots \quad z_{bfl} \quad z_{bfr} \quad z_{wfl} \quad z_{wfr} \quad z_{brl} \quad z_{brr} \quad z_{wrl} \quad z_{wrr}]' \tag{6}$$

$$U = \begin{bmatrix} F_{dfl} \\ F_{dfr} \\ F_{drl} \\ F_{drr} \end{bmatrix} \quad (7), \quad W = \begin{bmatrix} z_{ofl} \\ z_{ofr} \\ z_{orl} \\ z_{orr} \end{bmatrix} \quad (8)$$

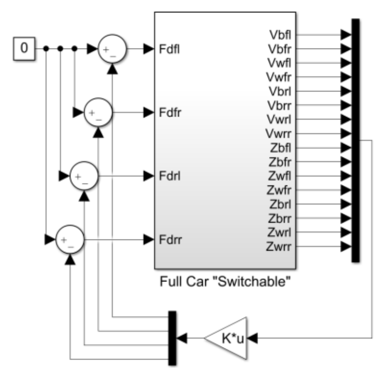


Fig. 10. LQR Simulink model for SDSS

$$A = \left[\begin{array}{cccccccc ccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k_{sfl}}{M_b} - \frac{{l_f}^2}{I_{by}}.k_{sfl} - \left(\frac{B}{2}\right)^2 \frac{k_{sfl}}{I_{bx}} & -\frac{k_{sfr}}{M_b} - \frac{{l_f}^2}{I_{by}}.k_{sfr} + \left(\frac{B}{2}\right)^2 \frac{k_{sfr}}{I_{bx}} & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k_{sfl}}{M_b} - \frac{{l_f}^2}{I_{by}}.k_{sfl} + \left(\frac{B}{2}\right)^2 \frac{k_{sfl}}{I_{bx}} & -\frac{k_{sfr}}{M_b} - \frac{{l_f}^2}{I_{by}}.k_{sfr} - \left(\frac{B}{2}\right)^2 \frac{k_{sfr}}{I_{bx}} & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{k_{sfl}}{M_{wfl}} & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{k_{sfr}}{M_{wfr}} & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k_{sfl}}{M_b} + \frac{l_f l_r}{I_{by}}.k_{sfl} - \left(\frac{B}{2}\right)^2 \frac{k_{sfl}}{I_{bx}} & -\frac{k_{sfr}}{M_b} + \frac{l_f l_r}{I_{by}}.k_{sfr} + \left(\frac{B}{2}\right)^2 \frac{k_{sfr}}{I_{bx}} & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k_{sfl}}{M_b} + \frac{l_f l_r}{I_{by}}.k_{sfl} + \left(\frac{B}{2}\right)^2 \frac{k_{sfl}}{I_{bx}} & -\frac{k_{sfr}}{M_b} + \frac{l_f l_r}{I_{by}}.k_{sfr} - \left(\frac{B}{2}\right)^2 \frac{k_{sfr}}{I_{bx}} & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \ldots
\end{array}\right.$$

$$
\begin{matrix}
\frac{k_{sfl}}{M_b}+\frac{{l_f}^2}{I_{by}}.k_{sfl}+\left(\frac{B}{2}\right)^2\frac{k_{sfl}}{I_{bx}} & \frac{k_{sfr}}{M_b}+\frac{{l_f}^2}{I_{by}}.k_{sfr}-\left(\frac{B}{2}\right)^2\frac{k_{sfr}}{I_{bx}} & -\frac{k_{srl}}{M_b}+\frac{l_f l_r}{I_{by}}.k_{srl}+\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & \dots \\
\frac{k_{sfl}}{M_b}+\frac{{l_f}^2}{I_{by}}.k_{sfl}-\left(\frac{B}{2}\right)^2\frac{k_{sfl}}{I_{bx}} & \frac{k_{sfr}}{M_b}+\frac{{l_f}^2}{I_{by}}.k_{sfr}+\left(\frac{B}{2}\right)^2\frac{k_{sfr}}{I_{bx}} & -\frac{k_{srl}}{M_b}+\frac{l_f l_r}{I_{by}}.k_{srl}-\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & \dots \\
-\frac{k_{sfl}+k_{tfl}}{M_{wfl}} & 0 & 0 & \dots \\
0 & -\frac{k_{sfr}+k_{tfr}}{M_{wfr}} & 0 & \dots \\
\frac{k_{sfl}}{M_b}-\frac{l_f l_r}{I_{by}}.k_{sfl}+\left(\frac{B}{2}\right)^2\frac{k_{sfl}}{I_{bx}} & \frac{k_{sfr}}{M_b}-\frac{l_f l_r}{I_{by}}.k_{sfr}-\left(\frac{B}{2}\right)^2\frac{k_{sfr}}{I_{bx}} & -\frac{k_{srl}}{M_b}-\frac{{l_r}^2}{I_{by}}.k_{srl}+\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & \dots \\
\frac{k_{sfl}}{M_b}-\frac{l_f l_r}{I_{by}}.k_{sfl}-\left(\frac{B}{2}\right)^2\frac{k_{sfl}}{I_{bx}} & \frac{k_{sfr}}{M_b}-\frac{l_f l_r}{I_{by}}.k_{sfr}+\left(\frac{B}{2}\right)^2\frac{k_{sfr}}{I_{bx}} & -\frac{k_{srl}}{M_b}-\frac{{l_r}^2}{I_{by}}.k_{srl}-\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & \dots \\
0 & 0 & \frac{k_{srl}}{M_{wrl}} & \dots \\
0 & 0 & 0 & \dots \\
0 & 0 & 0 & \dots \\
0 & 0 & 0 & \dots \\
0 & 0 & 0 & \dots \\
0 & 0 & 0 & \dots \\
0 & 0 & 0 & \dots \\
0 & 0 & 0 & \dots \\
0 & 0 & 0 & \dots \\
0 & 0 & 0 & \dots
\end{matrix}
$$

$$
\left.\begin{matrix}
-\frac{k_{srr}}{M_b}+\frac{l_f l_r}{I_{by}}.k_{srr}-\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} & \frac{k_{srl}}{M_b}-\frac{l_f l_r}{I_{by}}.k_{srl}-\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & \frac{k_{srr}}{M_b}-\frac{l_f l_r}{I_{by}}.k_{srr}+\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} \\
-\frac{k_{srr}}{M_b}+\frac{l_f l_r}{I_{by}}.k_{srr}+\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} & \frac{k_{srl}}{M_b}-\frac{l_f l_r}{I_{by}}.k_{srl}+\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & \frac{k_{srr}}{M_b}-\frac{l_f l_r}{I_{by}}.k_{srr}-\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
-\frac{k_{srr}}{M_b}-\frac{{l_r}^2}{I_{by}}.k_{srr}-\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} & \frac{k_{srl}}{M_b}+\frac{{l_r}^2}{I_{by}}.k_{srl}-\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & \frac{k_{srr}}{M_b}+\frac{{l_r}^2}{I_{by}}.k_{srr}+\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} \\
-\frac{k_{srr}}{M_b}-\frac{{l_r}^2}{I_{by}}.k_{srr}+\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} & \frac{k_{srl}}{M_b}+\frac{{l_r}^2}{I_{by}}.k_{srl}+\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & \frac{k_{srr}}{M_b}+\frac{{l_r}^2}{I_{by}}.k_{srr}-\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} \\
0 & -\frac{k_{srl}+k_{trl}}{M_{wrl}} & 0 \\
\frac{k_{srr}}{M_{wrr}} & 0 & -\frac{k_{srr}+k_{trr}}{M_{wrr}} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{matrix}\right] \quad (9)
$$

$$
B=\begin{bmatrix}
\frac{1}{M_b}+\frac{{l_f}^2}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}+\frac{{l_f}^2}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}-\frac{{l_f}^2}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}-\frac{{l_f}^2}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} \\
\frac{1}{M_b}+\frac{{l_f}^2}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}+\frac{{l_f}^2}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}-\frac{{l_f}^2}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}-\frac{{l_f}^2}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} \\
-\frac{1}{M_{wfl}} & 0 & 0 & 0 \\
0 & -\frac{1}{M_{wfr}} & 0 & 0 \\
\frac{1}{M_b}-\frac{l_f l_r}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}-\frac{l_f l_r}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}+\frac{l_f l_r}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}+\frac{l_f l_r}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} \\
\frac{1}{M_b}-\frac{l_f l_r}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}-\frac{l_f l_r}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}+\frac{l_f l_r}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}+\frac{l_f l_r}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} \\
0 & 0 & -\frac{1}{M_{wrl}} & 0 \\
0 & 0 & 0 & -\frac{1}{M_{wrr}} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

(10)

$$
D=\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{k_{tfl}}{M_{wfl}} & 0 & 0 & 0 \\
0 & \frac{k_{tfr}}{M_{wfr}} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \frac{k_{trl}}{M_{wrl}} & 0 \\
0 & 0 & 0 & \frac{k_{trr}}{M_{wrr}} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \quad (11)
$$

$$
C=\begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{bmatrix} \quad (12)
$$

Utilizing LQR function while defining the matrixes Q and R in MATLAB the proper set of gains can be found as follows;

$$
K\_SD = 1e3\begin{bmatrix}
1.8758 & 0.742 & -.0003 & .0044 & .7109 & -0.8299 & -0.0182 & 0.0153 & \dots \\
0.7413 & 1.8765 & 0.0044 & -0.0003 & -0.8292 & 0.7101 & 0.0153 & -0.0182 & \dots \\
0.7580 & -0.6162 & 0.0182 & -0.0175 & 2.2069 & 0.3310 & -0.0389 & 0.0436 & \dots \\
-0.6162 & 0.7580 & -0.0175 & 0.0182 & 0.3310 & 2.2069 & 0.0436 & -0.0389 & \dots
\end{bmatrix}
$$

$$
\left.\begin{matrix}
-0.9345 & 3.2949 & 0.0776 & -0.0757 & 7.0689 & -6.9462 & -0.2428 & 0.2375 \\
3.2949 & -0.9345 & -0.0757 & 0.0776 & -6.9462 & 7.0689 & 0.2375 & -0.2428 \\
-2.537 & 2.6526 & -0.0612 & 0.053 & 9.5507 & -7.576 & -0.4767 & 0.4776 \\
2.6526 & -2.537 & 0.0530 & -0.0612 & -7.576 & 9.5507 & 0.4776 & -0.4767
\end{matrix}\right]
$$

(13)

4) *LQR for ASS*

Fig. 11 shows the LQR Simulink model for the SDSS. Taking the state variables as follows:

$$
X=[\dot{z}_{bfl} \quad \dot{z}_{bfr} \quad \dot{z}_{wfl} \quad \dot{z}_{wfr} \quad \dot{z}_{brl} \quad \dot{z}_{brr} \quad \dot{z}_{wrl} \quad \dot{z}_{wrr} \quad \dots \quad z_{bfl} \quad z_{bfr} \quad z_{wfl} \quad z_{wfr} \quad z_{brl} \quad z_{brr} \quad z_{wrl} \quad z_{wrr}]' \quad (14)
$$

$$
U = \begin{bmatrix} F_{dfl} \\ F_{dfr} \\ F_{drl} \\ F_{drr} \end{bmatrix} \quad (15), \qquad W = \begin{bmatrix} z_{ofl} \\ z_{ofr} \\ z_{orl} \\ z_{orr} \end{bmatrix} \quad (16)
$$

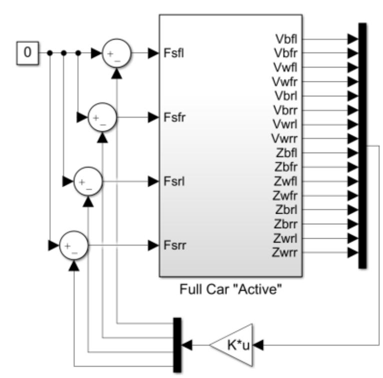


Fig. 11. LQR Simulink model for the ASS

$$A=\begin{bmatrix}
-\frac{c_{sfl}}{M_b}-\frac{l_f^2}{I_{by}}.c_{sfl}-\left(\frac{B}{2}\right)^2\frac{c_{sfl}}{I_{bx}} & -\frac{c_{sfr}}{M_b}-\frac{l_f^2}{I_{by}}.c_{sfr}+\left(\frac{B}{2}\right)^2\frac{c_{sfr}}{I_{bx}} & \frac{c_{sfl}}{M_b}+\frac{l_f^2}{I_{by}}.c_{sfl}+\left(\frac{B}{2}\right)^2\frac{c_{sfl}}{I_{bx}} & \frac{c_{sfr}}{M_b}+\frac{l_f^2}{I_{by}}.c_{sfr}-\left(\frac{B}{2}\right)^2\frac{c_{sfr}}{I_{bx}} & -\frac{c_{srl}}{M_b}+\frac{l_fl_r}{I_{by}}.c_{srl}+\left(\frac{B}{2}\right)^2\frac{c_{srl}}{I_{bx}} & -\frac{c_{srr}}{M_b}+\frac{l_fl_r}{I_{by}}.c_{srr}-\left(\frac{B}{2}\right)^2\frac{c_{srr}}{I_{bx}} & \frac{c_{srl}}{M_b}-\frac{l_fl_r}{I_{by}}.c_{srl}-\left(\frac{B}{2}\right)^2\frac{c_{srl}}{I_{bx}} & \frac{c_{srr}}{M_b}-\frac{l_fl_r}{I_{by}}.c_{srr}+\left(\frac{B}{2}\right)^2\frac{c_{srr}}{I_{bx}} & -\frac{k_{sfl}}{M_b}-\frac{l_f^2}{I_{by}}.k_{sfl}-\left(\frac{B}{2}\right)^2\frac{k_{sfl}}{I_{bx}} & -\frac{k_{sfr}}{M_b}-\frac{l_f^2}{I_{by}}.k_{sfr}+\left(\frac{B}{2}\right)^2\frac{k_{sfr}}{I_{bx}} & \frac{k_{sfl}}{M_b}+\frac{l_f^2}{I_{by}}.k_{sfl}+\left(\frac{B}{2}\right)^2\frac{k_{sfl}}{I_{bx}} & \frac{k_{sfr}}{M_b}+\frac{l_f^2}{I_{by}}.k_{sfr}-\left(\frac{B}{2}\right)^2\frac{k_{sfr}}{I_{bx}} & -\frac{k_{srl}}{M_b}+\frac{l_fl_r}{I_{by}}.k_{srl}+\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & -\frac{k_{srr}}{M_b}+\frac{l_fl_r}{I_{by}}.k_{srr}-\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} & \frac{k_{srl}}{M_b}-\frac{l_fl_r}{I_{by}}.k_{srl}-\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & \frac{k_{srr}}{M_b}-\frac{l_fl_r}{I_{by}}.k_{srr}+\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} \\
-\frac{c_{sfl}}{M_b}-\frac{l_f^2}{I_{by}}.c_{sfl}+\left(\frac{B}{2}\right)^2\frac{c_{sfl}}{I_{bx}} & -\frac{c_{sfr}}{M_b}-\frac{l_f^2}{I_{by}}.c_{sfr}-\left(\frac{B}{2}\right)^2\frac{c_{sfr}}{I_{bx}} & \frac{c_{sfl}}{M_b}+\frac{l_f^2}{I_{by}}.c_{sfl}-\left(\frac{B}{2}\right)^2\frac{c_{sfl}}{I_{bx}} & \frac{c_{sfr}}{M_b}+\frac{l_f^2}{I_{by}}.c_{sfr}+\left(\frac{B}{2}\right)^2\frac{c_{sfr}}{I_{bx}} & -\frac{c_{srl}}{M_b}+\frac{l_fl_r}{I_{by}}.c_{srl}-\left(\frac{B}{2}\right)^2\frac{c_{srl}}{I_{bx}} & -\frac{c_{srr}}{M_b}+\frac{l_fl_r}{I_{by}}.c_{srr}+\left(\frac{B}{2}\right)^2\frac{c_{srr}}{I_{bx}} & \frac{c_{srl}}{M_b}-\frac{l_fl_r}{I_{by}}.c_{srl}+\left(\frac{B}{2}\right)^2\frac{c_{srl}}{I_{bx}} & \frac{c_{srr}}{M_b}-\frac{l_fl_r}{I_{by}}.c_{srr}-\left(\frac{B}{2}\right)^2\frac{c_{srr}}{I_{bx}} & -\frac{k_{sfl}}{M_b}-\frac{l_f^2}{I_{by}}.k_{sfl}+\left(\frac{B}{2}\right)^2\frac{k_{sfl}}{I_{bx}} & -\frac{k_{sfr}}{M_b}-\frac{l_f^2}{I_{by}}.k_{sfr}-\left(\frac{B}{2}\right)^2\frac{k_{sfr}}{I_{bx}} & \frac{k_{sfl}}{M_b}+\frac{l_f^2}{I_{by}}.k_{sfl}-\left(\frac{B}{2}\right)^2\frac{k_{sfl}}{I_{bx}} & \frac{k_{sfr}}{M_b}+\frac{l_f^2}{I_{by}}.k_{sfr}+\left(\frac{B}{2}\right)^2\frac{k_{sfr}}{I_{bx}} & -\frac{k_{srl}}{M_b}+\frac{l_fl_r}{I_{by}}.k_{srl}-\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & -\frac{k_{srr}}{M_b}+\frac{l_fl_r}{I_{by}}.k_{srr}+\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} & \frac{k_{srl}}{M_b}-\frac{l_fl_r}{I_{by}}.k_{srl}+\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & \frac{k_{srr}}{M_b}-\frac{l_fl_r}{I_{by}}.k_{srr}-\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} \\
\frac{c_{sfl}}{M_{wfl}} & 0 & -\frac{c_{sfl}}{M_{wfl}} & 0 & 0 & 0 & 0 & 0 & \frac{k_{sfl}}{M_{wfl}} & 0 & -\frac{k_{sfl}+k_{tfl}}{M_{wfl}} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{c_{sfr}}{M_{wfr}} & 0 & -\frac{c_{sfr}}{M_{wfr}} & 0 & 0 & 0 & 0 & 0 & \frac{k_{sfr}}{M_{wfr}} & 0 & -\frac{k_{sfr}+k_{tfr}}{M_{wfr}} & 0 & 0 & 0 & 0 \\
-\frac{c_{sfl}}{M_b}+\frac{l_fl_r}{I_{by}}.c_{sfl}-\left(\frac{B}{2}\right)^2\frac{c_{sfl}}{I_{bx}} & -\frac{c_{sfr}}{M_b}+\frac{l_fl_r}{I_{by}}.c_{sfr}+\left(\frac{B}{2}\right)^2\frac{c_{sfr}}{I_{bx}} & \frac{c_{sfl}}{M_b}-\frac{l_fl_r}{I_{by}}.c_{sfl}+\left(\frac{B}{2}\right)^2\frac{c_{sfl}}{I_{bx}} & \frac{c_{sfr}}{M_b}-\frac{l_fl_r}{I_{by}}.c_{sfr}-\left(\frac{B}{2}\right)^2\frac{c_{sfr}}{I_{bx}} & -\frac{c_{srl}}{M_b}-\frac{l_r^2}{I_{by}}.c_{srl}+\left(\frac{B}{2}\right)^2\frac{c_{srl}}{I_{bx}} & -\frac{c_{srr}}{M_b}-\frac{l_r^2}{I_{by}}.c_{srr}-\left(\frac{B}{2}\right)^2\frac{c_{srr}}{I_{bx}} & \frac{c_{srl}}{M_b}+\frac{l_r^2}{I_{by}}.c_{srl}-\left(\frac{B}{2}\right)^2\frac{c_{srl}}{I_{bx}} & \frac{c_{srr}}{M_b}+\frac{l_r^2}{I_{by}}.c_{srr}+\left(\frac{B}{2}\right)^2\frac{c_{srr}}{I_{bx}} & -\frac{k_{sfl}}{M_b}+\frac{l_fl_r}{I_{by}}.k_{sfl}-\left(\frac{B}{2}\right)^2\frac{k_{sfl}}{I_{bx}} & -\frac{k_{sfr}}{M_b}+\frac{l_fl_r}{I_{by}}.k_{sfr}+\left(\frac{B}{2}\right)^2\frac{k_{sfr}}{I_{bx}} & \frac{k_{sfl}}{M_b}-\frac{l_fl_r}{I_{by}}.k_{sfl}+\left(\frac{B}{2}\right)^2\frac{k_{sfl}}{I_{bx}} & \frac{k_{sfr}}{M_b}-\frac{l_fl_r}{I_{by}}.k_{sfr}-\left(\frac{B}{2}\right)^2\frac{k_{sfr}}{I_{bx}} & -\frac{k_{srl}}{M_b}-\frac{l_r^2}{I_{by}}.k_{srl}+\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & -\frac{k_{srr}}{M_b}-\frac{l_r^2}{I_{by}}.k_{srr}-\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} & \frac{k_{srl}}{M_b}+\frac{l_r^2}{I_{by}}.k_{srl}-\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & \frac{k_{srr}}{M_b}+\frac{l_r^2}{I_{by}}.k_{srr}+\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} \\
-\frac{c_{sfl}}{M_b}+\frac{l_fl_r}{I_{by}}.c_{sfl}+\left(\frac{B}{2}\right)^2\frac{c_{sfl}}{I_{bx}} & -\frac{c_{sfr}}{M_b}+\frac{l_fl_r}{I_{by}}.c_{sfr}-\left(\frac{B}{2}\right)^2\frac{c_{sfr}}{I_{bx}} & \frac{c_{sfl}}{M_b}-\frac{l_fl_r}{I_{by}}.c_{sfl}-\left(\frac{B}{2}\right)^2\frac{c_{sfl}}{I_{bx}} & \frac{c_{sfr}}{M_b}-\frac{l_fl_r}{I_{by}}.c_{sfr}+\left(\frac{B}{2}\right)^2\frac{c_{sfr}}{I_{bx}} & -\frac{c_{srl}}{M_b}-\frac{l_r^2}{I_{by}}.c_{srl}-\left(\frac{B}{2}\right)^2\frac{c_{srl}}{I_{bx}} & -\frac{c_{srr}}{M_b}-\frac{l_r^2}{I_{by}}.c_{srr}+\left(\frac{B}{2}\right)^2\frac{c_{srr}}{I_{bx}} & \frac{c_{srl}}{M_b}+\frac{l_r^2}{I_{by}}.c_{srl}+\left(\frac{B}{2}\right)^2\frac{c_{srl}}{I_{bx}} & \frac{c_{srr}}{M_b}+\frac{l_r^2}{I_{by}}.c_{srr}-\left(\frac{B}{2}\right)^2\frac{c_{srr}}{I_{bx}} & -\frac{k_{sfl}}{M_b}+\frac{l_fl_r}{I_{by}}.k_{sfl}+\left(\frac{B}{2}\right)^2\frac{k_{sfl}}{I_{bx}} & -\frac{k_{sfr}}{M_b}+\frac{l_fl_r}{I_{by}}.k_{sfr}-\left(\frac{B}{2}\right)^2\frac{k_{sfr}}{I_{bx}} & \frac{k_{sfl}}{M_b}-\frac{l_fl_r}{I_{by}}.k_{sfl}-\left(\frac{B}{2}\right)^2\frac{k_{sfl}}{I_{bx}} & \frac{k_{sfr}}{M_b}-\frac{l_fl_r}{I_{by}}.k_{sfr}+\left(\frac{B}{2}\right)^2\frac{k_{sfr}}{I_{bx}} & -\frac{k_{srl}}{M_b}-\frac{l_r^2}{I_{by}}.k_{srl}-\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & -\frac{k_{srr}}{M_b}-\frac{l_r^2}{I_{by}}.k_{srr}+\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} & \frac{k_{srl}}{M_b}+\frac{l_r^2}{I_{by}}.k_{srl}+\left(\frac{B}{2}\right)^2\frac{k_{srl}}{I_{bx}} & \frac{k_{srr}}{M_b}+\frac{l_r^2}{I_{by}}.k_{srr}-\left(\frac{B}{2}\right)^2\frac{k_{srr}}{I_{bx}} \\
0 & 0 & 0 & 0 & \frac{c_{srl}}{M_{wrl}} & 0 & -\frac{c_{srl}}{M_{wrl}} & 0 & 0 & 0 & 0 & 0 & \frac{k_{srl}}{M_{wrl}} & 0 & -\frac{k_{srl}+k_{trl}}{M_{wrl}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{c_{srr}}{M_{wrr}} & 0 & -\frac{c_{srr}}{M_{wrr}} & 0 & 0 & 0 & 0 & 0 & \frac{k_{srr}}{M_{wrr}} & 0 & -\frac{k_{srr}+k_{trr}}{M_{wrr}} \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{17}$$

$$B=\begin{bmatrix}
\frac{1}{M_b}+\frac{l_f^2}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}+\frac{l_f^2}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}-\frac{l_f^2}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}-\frac{l_f^2}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} \\
\frac{1}{M_b}+\frac{l_f^2}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}+\frac{l_f^2}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}-\frac{l_f^2}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}-\frac{l_f^2}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} \\
-\frac{1}{M_{wfl}} & 0 & 0 & 0 \\
0 & -\frac{1}{M_{wfr}} & 0 & 0 \\
\frac{1}{M_b}-\frac{l_fl_r}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}-\frac{l_fl_r}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}+\frac{l_fl_r}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}+\frac{l_fl_r}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} \\
\frac{1}{M_b}-\frac{l_fl_r}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}-\frac{l_fl_r}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}+\frac{l_fl_r}{I_{by}}-\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} & \frac{1}{M_b}+\frac{l_fl_r}{I_{by}}+\left(\frac{B}{2}\right)^2\frac{1}{I_{bx}} \\
0 & 0 & -\frac{1}{M_{wrl}} & 0 \\
0 & 0 & 0 & -\frac{1}{M_{wrr}} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \tag{18}$$

$$D=\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{k_{tfl}}{M_{wfl}} & 0 & 0 & 0 \\
0 & \frac{k_{tfr}}{M_{wfr}} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \frac{k_{trl}}{M_{wrl}} & 0 \\
0 & 0 & 0 & \frac{k_{trr}}{M_{wrr}} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \tag{19}$$

$$C = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \tag{20}$$

Utilizing LQR function while defining the matrixes Q and R in MATLAB the proper set of gains can be found as follows;

$$K\_A = 1e4 \begin{bmatrix} 0.8785 & 0.2735 & 0.0189 & 0.0017 & 0.1384 & -.1423 & -0.0105 & 0.0101 & \dots \\ 0.5013 & 0.4374 & 0.0018 & 0.0148 & -0.441 & 0.2997 & 0.0054 & -0.01 & \dots \\ 0.28 & -0.032 & 0.0287 & -0.0194 & 1.09 & -0.0067 & -0.0131 & 0.0431 & \dots \\ -0.4239 & 0.2699 & 0.0254 & 0.0186 & 0.1399 & 0.6904 & 0.0345 & -0.0169 & \dots \end{bmatrix}$$

$$\begin{matrix} 3.8046 & 0.9105 & -2.4972 & -1.0414 & 1.307 & -0.8812 & 0.979 & -0.6247 \\ 1.983 & 1.5042 & -1.0814 & -2.3368 & -3.1268 & 1.7283 & 1.7283 & 1.0042 \\ 1.5724 & -0.1844 & -1.1338 & 1.016 & 7.6398 & -0.7333 & -1.2229 & -3.3626 \\ -2.0968 & 1.1629 & 1.0737 & -0.9118 & 0.1872 & 3.3751 & -3.0519 & -1.0849 \end{matrix}\Bigg]$$

(21)

V. Power Calculation

The power losses in the vehicle suspension system, is divided into three elements [16]:

- The fluctuating power in the spring.
- The P_{Diss} by the damper or actuator.
- The P_{Dem} for the actuator.

These power requirements represent the main part of the load on the vehicle's power supply. They can be calculated as follows:

$$Power = Force \times Relative\ Velocity \tag{22}$$

The fluctuating powers of the passive spring and tire have not been taken into consideration due to its low importance and impact as, their effect on the power supply is limited to the initial period of acceleration and the results calculated indicated that the differences in RMS values between all the suspension systems are limited [18].

VI. Results

1) Comparison of SDSS with LQR and FLC strategies in terms of ride performance

The percentage of improvements of body acceleration for SDSS using LQR and FLC strategies over the PSS are shown in Fig. 12. It is clear that, the SDSS with FLC gives better ride comfort compared with SDSS with LQR and PSS. The greatest improvement is gained for body acceleration at front left using the FLC strategy. Fig. 13 compares the power spectral density (PSD) results of LQR and FLC strategies for the SDSS with the PSS results in terms of body acceleration, suspension work space (SWS) and dynamic tire load (DTL).

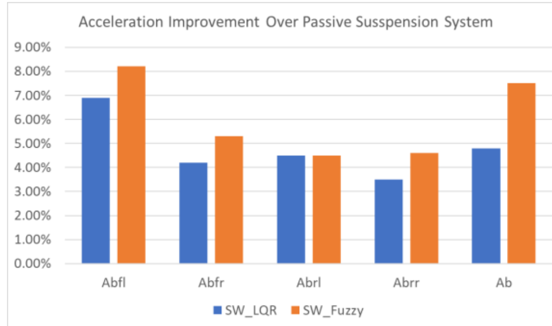
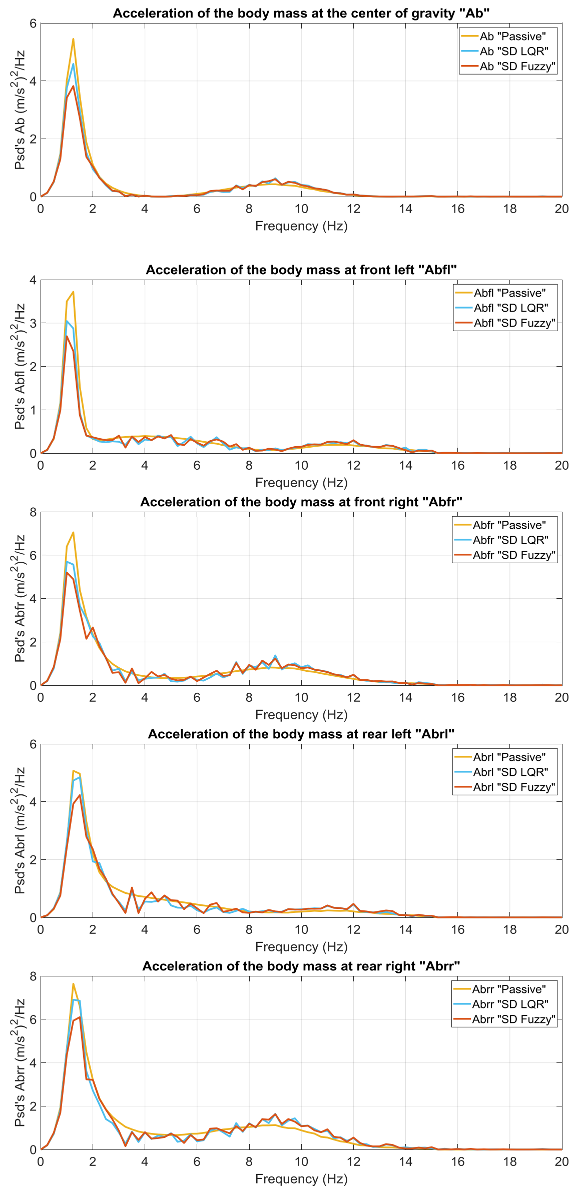


Fig. 12. Comparison of body accelerations for SDSS with LQR and FLC strategies relative to the PSS.



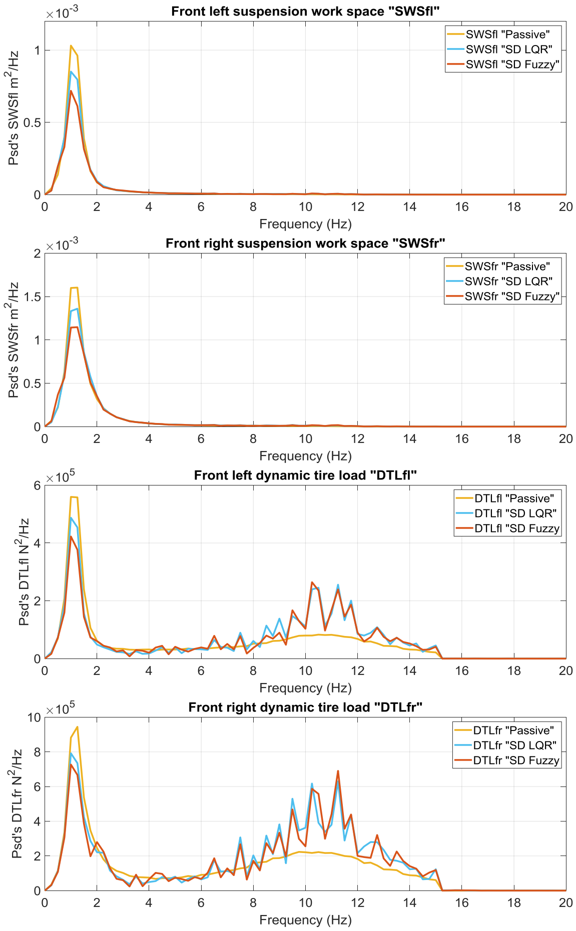


Fig. 13. PSD Comparison for SDSS with LQR and FLC strategies relative to the PSS.

2) *Comparison of ASS with LQR and FLC strategies in terms of ride performance*

The body acceleration in terms of RMS for the ASS using FLC strategy compared with the ASS with LQR and PSS are shown in Fig. 14. It is clear that the ASS with FLC gives the best ride comfort compared with ASS with LQR and PSS. These significant improvements are appeared clearly for roll, pitch, front, and rear body accelerations using FLC strategy. Fig. 15 compares the PSD results of LQR and FLC strategies for the ASS with the PSS results in terms of body acceleration, body roll, pitch movement, SWS and DTL.

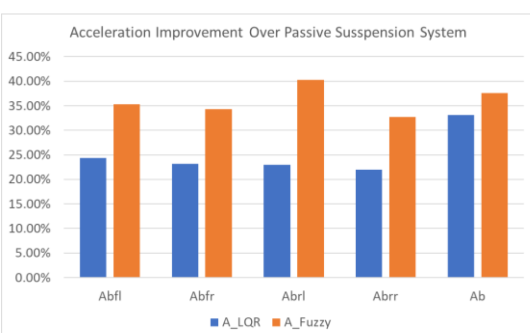
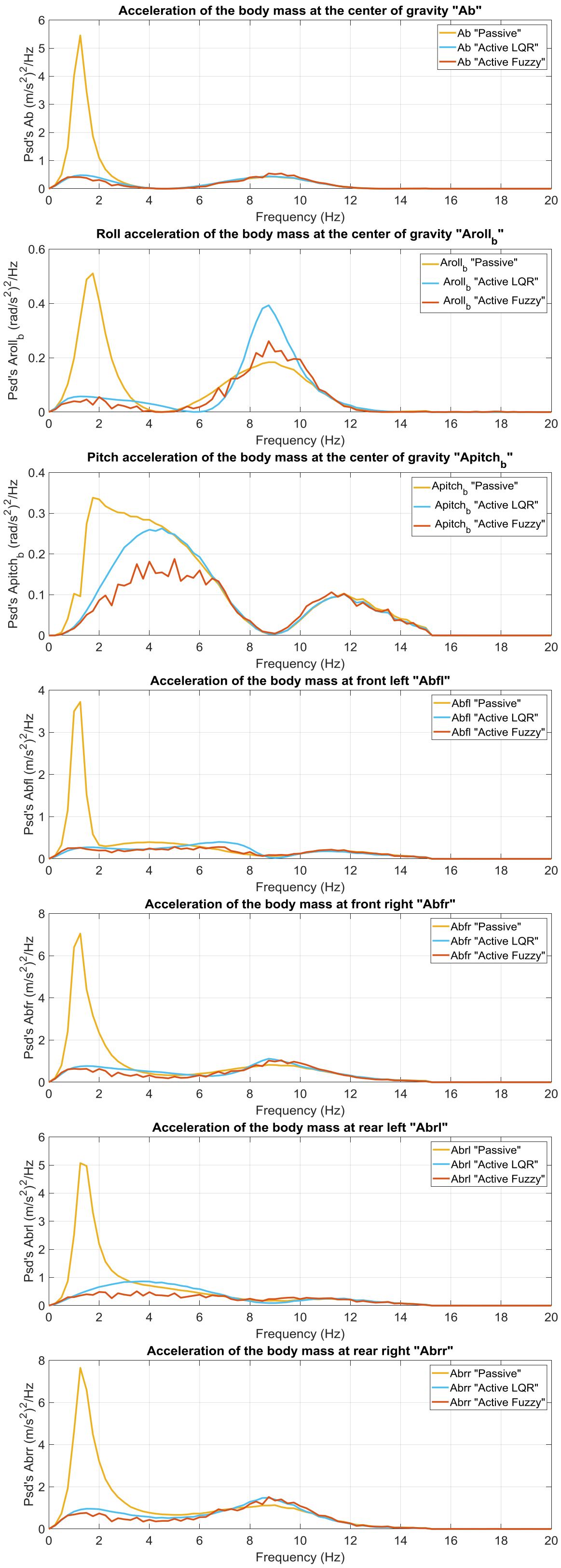


Fig. 14. Ride comfort improvement of ASS over the PSS using LQR and FLC strategies.



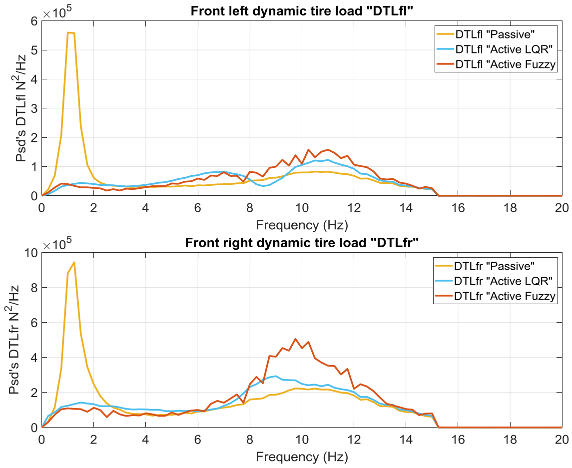


Fig. 15. PSD Comparison for ASS with LQR and FLC strategies relative to the PSS.

3) *SDSS and ASS with LQR and FLC strategies*

The ride performance criteria for three-setting SDSS and ASS compared with the PSS is shown in Table IV. The results the three-setting SDSSand ASS using LQR and FLC strategies are expressed in terms of the percentage improvements over the PSS. It is clear that the range of the percentage reductions of body acceleration at front left for SDSS and active systems using FLC are 8.2% and 35.3% respectively. The ASS with FLC gives the best ride performance compared with ASS with LQR and three-setting SDSS with FLC and LQR strategies.

TABLE IV
RMS Comparison Between Passive, SD and ASS

	SD LQR	**SD Fuzzy**	**Active LQR**	**Active Fuzzy**
	Improv.	Improv.	Improv.	Improv.
A_{bfl}	6.9%	8.2%	24.4%	35.3%
A_{bfr}	4.2%	5.3%	23.2%	34.3%
A_{brl}	4.5%	4.5%	23.0%	40.3%
A_{brr}	3.5%	4.6%	22.0%	32.7%
A_b	4.8%	7.5%	33.1%	37.6%
$\ddot{\theta}_b$	-4.6%	-1.2%	10.4%	69.4%
$\ddot{\phi}_b$	5.8%	3.0%	9.7%	25.6%
SWS_{fl}	3.4%	9.9%	12.8%	5.9%
SWS_{fr}	2.5%	5.4%	-5.2%	-14.2%
SWS_{rl}	-4.5%	-0.5%	10.1%	3.1%
SWS_{rr}	-4.1%	-1.9%	-9.9%	-25.0%
DTL_{fl}	-7.9%	-5.6%	6.9%	2.7%
DTL_{fr}	-8.4%	-7.9%	4.2%	-4.0%
DTL_{rl}	-11.5%	-9.0%	11.4%	4.6%
DTL_{rr}	-13.9%	-13.9%	-2.0%	-10.8%

4) Systems comparisons in terms of power requirement.

a. *SDSS with LQR and FLC Strategies*

The P_{Diss} for three-setting SDSS with LQR and FLC strategies are calculated in Table V. Looking first at the P_{Diss} results showed that the difference between these systems is up to 5% at the front and rear. The P_{Diss} of three-setting SDSS for the rear left with LQR and FLC at a vehicle speed of 90 km/hr, are 109.4 W and 109.2 W, while the P_{Diss} for the same system at rear right are 231.3 W and 233.9 W respectively. It is clear that the three-setting SDSS with FLC gives better ride performance compared with the same system with LQR. Furthermore, the P_{Diss} of SDSS with FLC is less by 5% compared with the same system with LQR at the front and rear. This SDSS is an attractive choice as a commercial vehicle suspension system as it is cheaper and simpler in comparison with other ASS.

TABLE V
P_{Diss} of SDSS with LQR and FLC

	SD LQR, Watt	**SD Fuzzy, Watt**
	Dissipate	Dissipate
Front Left Damper	96.16	94.54
Front Right Damper	250.3	253.7
Rear Left Damper	109.4	109.1
Rear Right Damper	231.3	233.9

Comparison between the time domain of three-setting SDSS with LQR and FLC at the front in terms of P_{Diss} using vehicle speed of 90 km/hr is shown in Fig. 16. It is clear that the peak values of the SDSS with LQR are higher compared with the SDSS with fuzzy; this is clearly seen in a time range between 1 to 5 seconds. However, this difference is depending on other factors e.g. road surface conditions. The fluctuating powers of the passive spring and tire have not been taken into consideration due to its low importance and impact as, their effect on the power supply is limited to the initial period of acceleration.

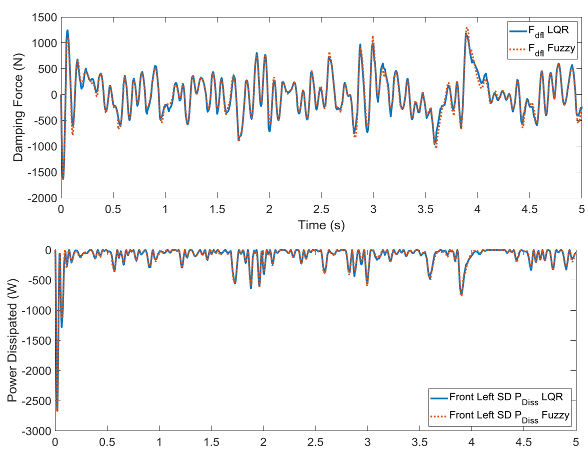


Fig. 16. Time histories of the P_{Diss} and damping force of the SDSS with LQR and FLC

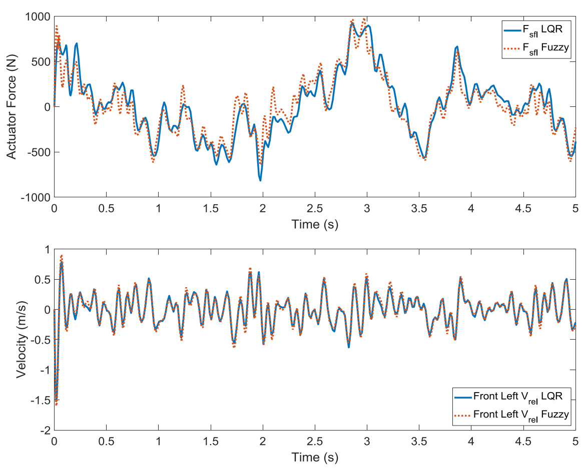
b. *Passive and ASS with LQR and FLC Strategies*

The $\overline{P}_{Dem}$ calculated assuming (i) that the components are ideal and (ii) that the spring in parallel with the actuator bears the static vehicle's weight. P_{Dem} and P_{Diss} of the ASS with LQR and FLC strategy are calculated in Table VI. It is clear that there are a few differences between the front and rear P_{Dem} for each control strategy used. The $\overline{P}_{Dem}$ of ASS for the rear left with LQR and FLC at a vehicle speed of 90 km/hr, are 25.63 W and 37.34 W, and for rear right are 68.34 W and 104.8 W respectively. However, these values are depending on other factors (e.g. Road surface conditions, road gradient, vehicle speed, and transmission efficiency).

TABLE VI
Power Requirements of ASS with LQR and FLC

	Active LQR		Active Fuzzy	
	Dissipate	Demand	Dissipate	Demand
Front Left Damper	88.55		98.01	
Front Right Damper	231.5		273.6	
Front Left Actuator	28.02	31.87	19.94	35.36
Front Right Actuator	70.89	65.34	55.13	96.56
Rear Left Damper	86.29		98.23	
Rear Right Damper	218		271	
Rear Left Actuator	30.54	25.63	20.64	37.34
Rear Right Actuator	55.06	68.34	41.55	104.8

Fig. 17 shows the time domain of actuator force, relative velocity, and P_{Dem} for the ASS. In this figure, it is clear that the peak values are much higher intermittent P_{Dem} for ASS compared with the mean value in Table VI. This is important from a practical point of view for the actuator system design.



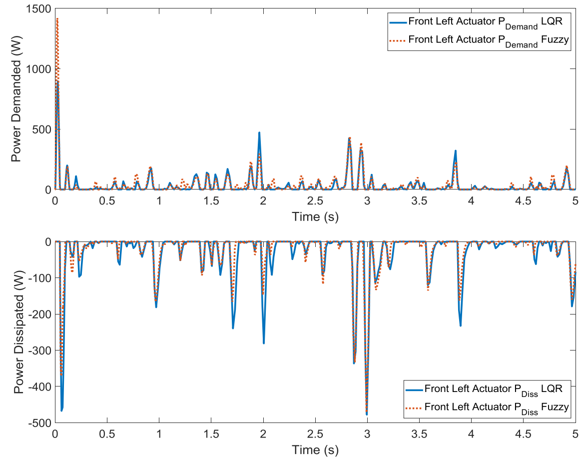


Fig. 17. Time histories of the actuator force, relative velocity, P_{Dem} and P_{Diss} of the active system

VII. Conclusion

Theoretical calculations based on the full-car model for the SDSS and ASS with LQR and FLC strategies have shown the following findings with respect to power requirements and performance.

1. The ASS with FLC offer the best overall performance compared with the ASS with LQR and SDSS with LQR and FLC strategies. However, the ASS involves extremely high costs and considerable practical complexity.

2. The three-setting SDSS with FLC gives better ride performance compared with the same system with LQR. Furthermore, this SDSS with FLC needs small P_{Dem}. This SDSS is an attractive choice as a commercial car suspension system as it is cheaper and simpler in comparison with other ASS.

3. The peak values of P_{Dem} for ASS using both strategies are much higher intermittent P_{Dem} compared with the mean value. This is important from a practical point of view for the actuator system design.

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NOTATIONS

A_b	Acceleration of the vehicle body at c.g.
A_{bfl}	Acceleration of the vehicle body at front left
A_{bfr}	Acceleration of the vehicle body at front right
A_{brl}	Acceleration of the vehicle body at rear left
A_{brr}	Acceleration of the vehicle body at rear right
B	Wheel track
c.g.	Center of gravity
C_h	Hard setting of the SD
C_m	Medium setting of the SD
C_s	Soft setting of the SD
C_{sfl}	Front left damper coefficient
C_{sfr}	Front right damper coefficient
C_{srl}	Rear left damper coefficient
C_{srr}	Rear right damper coefficient
DTL_{fl}	Dynamic tire load of the vehicle body at front left
DTL_{fr}	Dynamic tire load of the vehicle body at front right
DTL_{rl}	Dynamic tire load of the vehicle body at rear left
DTL_{rr}	Dynamic tire load of the vehicle body at rear right
f	Vector of functions
FA	Firefly algorithm
F_{dfl}	SD damping force at front left
F_{dfr}	SD damping force at front right
F_{drl}	SD damping force at rear left
F_{drr}	SD damping force at rear right
FIS	Fuzzy inference system
F_{sfl}	Active actuator force at front left
F_{sfr}	Active actuator force at front right
F_{srl}	Active actuator force at rear left
F_{srr}	Active actuator force at rear right
I_b	Pitch moment of inertia
IFL	Intelligent fuzzy logic
K_{sfl}	Front left spring stiffness
K_{sfr}	Front right spring stiffness
K_{srl}	Rear left spring stiffness
K_{srr}	Rear right spring stiffness
K_{tfl}	Front left tire spring stiffness
K_{tfr}	Front right tire spring stiffness
K_{trl}	Rear left tire spring stiffness
K_{trr}	Rear right tire spring stiffness
L	Wheelbase
L_f	Distance from the front axle to c.g.
LQR	Linear quadratic regulator
L_r	Distance from the rear axle to c.g.
M_b	Body mass
MFs	Membership functions

MISO	Multi input single output
MR	Magneto-rheological
M_{wfl}	Front left wheel mass
M_{wfr}	Front right wheel mass
M_{wrl}	Rear left wheel mass
M_{wrr}	Rear right wheel mass
N	Negative error
NL	Large negative error
NM	Medium negative error
NS	Small negative error
P	Positive error
PID	Proportional integral derivative
PL	Large positive error
PM	Medium positive error
PS	Small positive error
PSD	Power spectral density
PSO	Particle swarm optimization
r.m.s	Root mean square
SA	Semi-active
SD	Switchable damper
SWS_{fl}	Suspension work space of the vehicle body at front left
SWS_{fr}	Suspension work space of the vehicle body at front right
SWS_{rl}	Suspension work space of the vehicle body at rear left
SWS_{rr}	Suspension work space of the vehicle body at rear right
Δt	Delay between the front and rear wheel
T_d	Time delay of the SD
T_{th}	Threshold time of the SD
U	Vehicle speed
u_1	First input for fuzzy controller
u_2	Second input for fuzzy controller
x_b	Vehicle’s body vertical displacement
$\ddot{x}_b$	Vehicle’s body vertical acceleration
x_w	Wheel vertical displacement
$\ddot{x}_w$	Wheel vertical acceleration
y_1	First output of the fuzzy controller
Z_b	Vertical displacement of vehicle body at c.g.
Z_{bfl}	Vertical displacement of vehicle body at front left
Z_{bfr}	Vertical displacement of vehicle body at front right
Z_{brl}	Vertical displacement of vehicle body at rear left
Z_{brr}	Vertical displacement of vehicle body at rear right
Z, ZE	Zero error
Z_{ofl}	Road profile at front left
Z_{ofr}	Road profile at front right
Z_{orl}	Road profile at rear left
Z_{orr}	Road profile at rear right
Z_w	Wheel vertical displacement
Z_{wfl}	Front left wheel vertical displacement
Z_{wfr}	Front right wheel vertical displacement
Z_{wrl}	Rear left wheel vertical displacement
Z_{wrr}	Rear right wheel vertical displacement
Θ_b	Pitch angle of vehicle body around c.g.
$\ddot{\Theta}_b$	Pitch acceleration of the vehicle body around c.g.
$\ddot{\Phi}_b$	Roll acceleration of the vehicle body around c.g.